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The Arnold-Liouville theorem for contact geometry

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Given a Hamiltonian system on a $(2n)$-dimensional symplectic manifold (M, ω, H), on some situations it is possible to find an independent family of conserved quantities $(f_i)_{i=1}^n : M \to \mathbb{R}$. If this quantities commute with respect to the Poisson bracket, provided some regularity properties are satised, then the level sets of $(f_i)_i$ are a foliation of (M, ω) by Lagrangian submanifolds diffeomorphic to Abelian Lie groups $(T^k \times \mathbb{R}^{n-k})$. The Hamiltonian dynamics is tangent to this foliation. Moreover, one can find the so-called angle-action coordinates in which the equations of motion are linear with constant coefficients in each of the submanifolds. This is the Arnold-Liouville theorem (in the noncompact case). This theorem can be generalized in many directions. One possibility is to study what happens when we depart from symplectic geometry and we consider other types of Hamiltonian systems (e.g. time dependent, action dependent or homogeneous). Recently, we were able to extend this result to Liouville and contact Hamiltonian systems. Based on this findings, we propose an extension of the definition of the moment map for contact integrable systems.