Workshop Combinatorial Problems in Algebraic and Symplectic Geometry
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Ewald's Conjecture and integer points in algebraic and symplectic toric geometry

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This talk presents several open problems concerning integer points of polytopes arising in symplectic and algebraic geometry, and the solutions for some of them. Ewald's Conjecture from 1988 states that if P is a monotone n-polytope in $\mathbb{R}^n$ then the set $\mathbb{Z}^n \cap P \cap -P$ contains a unimodular basis of the lattice $\mathbb{Z}^n$. In 2009 Nill proposed a generalization of Ewald's Conjecture, which says that if P is an n-dimensional lattice smooth polytope in $\mathbb{R}^n$ then $\mathbb{Z}^n \cap P \cap -P$ contains a unimodular basis of $\mathbb{Z}^n$.

I will present proofs joint with Álvaro Pelayo of the following cases of these conjectures: Ewald's conjecture for n-polytopes which do not recursively contain unimodular triangles and Nill's conjecture for $n = 2$. Ewald's Conjecture is closely related to problems in toric geometry, and I will state the implications of our results in this context. A full version of this talk is available at arXiv:2310.10366.