SCHEDULE

	Monday 24 Oct	Tuesday 25 Oct	Wednesday 26 Oct	Thursday 27 Oct
9-10	9:00-10:00 **FHZ** course	9:00-10:00 **FHZ** course	9:00-10:00 **Irma Pallarés**	9:00-10:00 **FHZ** course
10-11	brief discussion	brief discussion	Posters/Coffee break	Posters/Coffee break
	10:15-11:15 **FHZ** course	10:15-11:15 **FHZ** course	10:30-12:00 **LNE** course	10:30-12:00 **LNE** course
11-12	Coffee break	Coffee break		
12-13	11:45-13:15 **MC** course	11:45-13:15 **LNE** course	12:00-13:30 **MC** course	12:00-13:00 **José Seade**
13-14				Closing
14-15	LUNCH TIME			
15-16	15:00-16:00 **Patrick Popescu-Pampu**	15:00-16:30 **MC** course	15:00-16:00 **FHZ** course	
16-17	Break		Break	
	16:30-17:30 **Bárbara Karolline de Lima Pereira**		16:30-18:00 **4 short contributed talks by junior participants**	
17-18				
			21:00 **Conference dinner**	

Courses

Course FHZ: "*Floer Homology towards the Zariski conjecture*"
by Javier Fernández de Bobadilla/Tomasz Pełka (BCAM)

ABSTRACT

In this course we will sketch our recent proof of equimultiplicity of families with constant Milnor number (joint with T. Pelka). We will put special emphasis in Floer theory constructions, and a substantial part of the course will be an introduction to Floer theory.

Prerequisites: I will assume familiarity with basic differential topology, algebraic topology and symplectic geometry, as for example in Chapter 5 of **[1]**. For students needing to build background on Differential and Algebraic Topology the following lectures, which will take place before the course may be of help:

http://www.bcamath.org/en/

[1] Audin-Damian, "Morse Theory and Floer Homology", Universitext.

Courses

Course MC: "*The Monodromy Conjecture*"

by Willem Veys (KU Leuven)

ABSTRACT

The monodromy conjecture is a mysterious open problem in singularity theory. It relates arithmetic and topological/geometric properties of a multivariate polynomial over the integers. The case of interest is when the zero set of the polynomial has singular points.

We first present some history and motivation, a proof in the case of two variables, a few partial results in higher dimension, and somewhat unexpected geometric theorems inspired by the conjecture.

Prerequisites: Basics of p-adic numbers (See e.g. sections 1.3 and 1.4 in **[1]**, or the main definitions and results of sections 2 and 3 in **[2]**).

[1] Viu-Sos, J., An introduction to p-adic and motivic integration, zeta functions and invariants of singularities. *p-adic analysis, arithmetic and singularities, 103–176, Contemp. Math., 778, 2022*. Lecture notes.

[2] Zúñiga-Galindo, W. A., p-adic analysis: a quick introduction. *p-adic analysis, arithmetic and singularities, 177–221, Contemp. Math., 778, 2022*. Lecture notes.

Courses

Course LNE: "*Lipschitz normal embedding on singular spaces*"

by Lorenzo Fantini (École Polythécnique de Paris)

ABSTRACT

Lipschitz geometry is a branch of singularity theory that studies a complex analytic germ $(X,0)\subset(\mathbb{C}^n,0)$ by equipping it with either one of two metrics: its outer metric, induced by the euclidean metric of the ambient space, and its inner metric, given by measuring the length of arcs on $(X,0)$.
Whenever those two metrics are equivalent up to a bi-Lipschitz homeomorphism, the germ is said to be Lipschitz normally embedded (LNE).

I will give an overview of this theory and then discuss the inner metric structure of a normal complex surface germ, as well as several geometric properties of LNE surface germs, criteria to prove that a surface germ is LNE, and an application to the so-called problem of polar exploration, that is the quest to determine the combinatorics of the generic polar curve of a complex surface from its topology.

There will be no particular prerequisites: some basic familiarity with undergraduate algebraic geometry will be sufficient.

Talks

Talk: "*The local tropicalization of splice type singularities*"

by Patrick Popescu-Pampu (Lab. Paul Painlevé, Univ. de Lille, France)

ABSTRACT

Splice type singularities are a huge extension of the class of Brieskorn-Pham-Hamm complete intersections. They were introduced by Neumann and Wahl around 2000. They are defined by explicite systems of equations, whose structures depend on special types of decorated trees, called splice diagrams. The links of those singularities realise all known integral homology sphere links of complex isolated complete intersections.

I will describe the local tropicalizations of splice type singularities and how this knowledge allows to prove that they are Newton non-degenerate complete intersections, which implies that each one of them may be resolved using a single toric morphism.

This is common work with Maria Angelica Cueto and Dmitry Stepanov.

Talk: "*The Bruce-Roberts Numbers of a Function on an Isolated Complete Intersection Singularity*"

by Bárbara Karolline de Lima Pereira (ICMC-USP, Brazil)

ABSTRACT

We give formulas for the Bruce-Roberts number $\mu_{BR}(f, X)$ and its relative version $\mu^{-}_{BR}(f, X)$ of a function f with respect to an ICIS $(\mathrm{X},0)$. We show that

$$\mu^{-}_{BR}(f, X) = \mu(f^{-1}(0) \cap X, 0) + \mu(X, 0) - \tau(X, 0),$$

where μ and τ are the Milnor and Tjurina numbers of the ICIS, respectively.
The formula for $\mu_{BR}(f, X)$is more complicated and also involves $\mu(f)$ and some lengths in terms of the ideals I_X and Jf. As a consequence the relation for the relative Bruce-Roberts number we show that $\mathrm{LC(X)}^{-}$ is Cohen-Macaulay. These results generalize our previous one, when $\$(X, 0)$ has codimension one and the results presented by Bruce and Roberts when it is weighted homogeneous of any codimension.

This is a joint work with J. J. Nuño-Ballesteros (Universitat de Valencia, SPAIN), B. Oréfice-Okamoto, (UFSCar, BRAZIL) and J.N. Tomazella, (UFSCar, BRAZIL).

Talks

Talk: "*The Brasselet-Schürmann-Yokura conjecture*"

by Irma Pallarés (K.U. Leuven, Belgium)

ABSTRACT

Brasselet, Schürmann, and Yokura formulated a conjecture in the theory of characteristic classes of singular spaces predicting the equality between the Hodge-Hirzebruch L-class and the Goresky-MacPherson L-class for compact complex algebraic varieties that are rational homology manifolds. This conjecture is the characteristic class version for rational homology manifolds of the Hodge index theorem that expresses the signature of compact complex algebraic manifolds in terms of their Hodge numbers. We prove the conjecture using the theory of mixed Hodge modules. This is a joint work with J. Fernández de Bobadilla and M. Saito. The particular case of projective varieties, can also be proved without using Hodge modules.

Talk: "*¿Qué tan trivial es un fibrado y por qué nos interesa saberlo?*"

by José Seade (UNAM, Mexico)

ABSTRACT

Hablaremos sobre diversos aspectos acerca de la trivialidad de fibrados vectoriales y aplicaciones en teoría de singularidades.

Short talks

Talk: "*All links are semiholomorphic*"

by Benjamin Bode (ICMAT, Spain)

ABSTRACT

A map $f : \mathbb{C}^2 \to \mathbb{C}$ is called a *semiholomorphic polynomial* if it can be written as a polynomial in complex variables u,v and the conjugate $\overline{v}$. Thus it is holomorphic with respect to one variable, but not necessarily with respect to the other. Semiholomorphic polynomials form an interesting family between the real and the complex setting.

Akbulut and King proved that *All knots are algebraic*, that is, every link in the 3-sphere arises as the link of a weakly isolated singularity of a real polynomial map $f : \mathbb{R}^4 \to \mathbb{R}^2$. In this short talk I will show that the statement is also true in the family of semiholomorphic polynomials.

Talk: "*Algebraic integrability of given genus*"

by Elvira Pérez Callejo (Universitat Jaume I, Spain)

ABSTRACT

The decision about algebraic integrability of a plane foliation is an open problem. We will show the existence of an algorithm that, under certain conditions, determines whether the foliation has a rational first integral of a given genus and, in the affirmative case, computes it. This algorithm uses the extension on a Hirzebruch surface and the reduction of the dicritical singularities of this foliation.

Short talks

Talk: "*Milnor fibre boundaries of non-isolated toric singularities*"

by András Sándor (Alfréd Rényi Institute of Mathematics, Hungary)

ABSTRACT

We know a lot about the Milnor fibre of isolated surface singularities and much less in the non-isolated case. We consider a particular family of non-isolated toric surface singularities that is created from cyclic quotient surface singularities X_σ.
We obtain a non-isolated singularity by deleting a family of monomials from the toric semigroup of X_σ. Then we define a toric deformation of this that gives us a Milnor fibre in turn. Finally, we describe the boundary of this Milnor fibre as a graph manifold.

Talk: "*The Euler obstruction of a map and its geometry*"

by Hellen Santana (ICMC- USP, Brazil)

ABSTRACT

We investigate the topological information captured by the Euler obstruction of a map-germ, $f : (X, 0) \to (\mathbb{C}^2, 0)$, where $(\mathrm{X},0)$ denote a germ of a complex d-equidimensional singular space, and its relation with the local Euler obstruction of the coordinate function.

Poster Session I

Poster: "*Deformation of frontals: a downstairs theory of Legendrian equivalence*"

by Christian Muñoz-Cabello (Universitat de València, Spain)

ABSTRACT

Frontals hypersurfaces are traditionally described as the image of a Legendrian map with singularities, and classified using diffeomorphisms that preserve the Legendrian fibres in the total space.

A frontal can also be described intrinsically as a C^∞ or analytic hypersurface admitting a global tangent hyperplane that varies smoothly relative to the base point.

In this joint work with J.J. Nuño-Ballesteros and R. Oset Sinha, we develop a intrinsic classification theory of frontals and showcase some results parallel to R. Thom and J. Mather's theory of smooth deformations.

Poster: "*Approximation results of Artin-Tougeron-type for general filtrations and for rings of smooth functions*"

by Alberto F. Boix (IMUVA - Universidad de Valladolid, Spain)

ABSTRACT

Various versions of Artin approximation are widely used in Algebraic/Analytic Geometry, Commutative Algebra and Singularity Theory; traditionally, the approximation statements were restricted to Noetherian rings and to filtrations by powers of ideals. The goal of this poster is to explain how to extend some of the classical approximation results both for general filtrations and for rings of smooth functions; for instance, our statements allow some immediate applications of Artin approximation to the study of non-isolated singularities of maps and schemes.

The content of this poster is based on joint work **[1]** with Genrich Belitskii and Dmitry Kerner (Ben Gurion University of the Negev, Beer Sheva, Israel).

[1] G. Belitskii, A. F. Boix, and D. Kerner. Approximation results of Artin-Tougeron-type for general filtrations and for C^r-equations. *J. Pure Appl. Algebra, 224(12):106431, 15, 2020.*

Poster Session I

Poster: "*Curvilinear Hilbert schemes and Algebraic Link Invariants of Plane Curve Singularities*"

by Ilaria Rossinelli (École Polytechnique Fédérale de Lausanne, Switzerland)

ABSTRACT

This poster aims to present some conjectures and results lying on the intersection of different topics in singularity theory, and tools and techniques coming from
the theory of p-adic integration to study invariants of singularities.

Given an isolated reduced plane curve singularity, we are naturally interested in studying its geometric and topological properties. The common approach makes use of mathematical objects associated to the curve singularity, and their invariants.

Two such objects one can decide to work with are the *algebraic link of the curve singularity* and the *punctual Hilbert scheme of the curve singularity*, which are very different constructions that seem to be surprisingly well-related, although in a way that is still quite obscure. Over the years many different flavours of invariants have been considered for links; however, their geometric and topological understanding in terms of the curve singularity is still very far.
This is why punctual Hilbert schemes come into play, since they seem to offer a fruitful geometric framework for the interpretation of such invariants.
In this direction we can find a theorem by Maulik and the Oblomkov-Rasmussen-Shende conjecture, that propose an interpretation at different depths of various link invariants in terms of punctual Hilbert schemes of the curve singularity and other classical invariants of the curve singularity.

In this poster, after introducing the setting mentioned above, we present a possible approach to the Oblomkov-Rasmussen-Shende conjecture based on the theory of p-adic integration.
Indeed, these particular integrals not only encode the number of solutions over certain finite fields of the given curve, but also capture the geometric behaviour of the singularity of the interested curve.
As a first step, we study the so-called *curvilinear Hilbert schemes* and some singularity invariants we build from them using these integration techniques, which seem to play an interesting role in the theory and will hopefully help to improve and deepen the understanding of the objects involved in the conjecture.

Poster Session I

Poster: "*On the generators of the value semigroup at infinity associated to a curve with only one place at infinity*"

by Carlos Galindo, Francisco Monserrat, Carlos-Jesús Moreno-Ávila y Julio-José Moyano-Fernández (Universitat Jaume I, Universitat Politècnica de València, Universidad de Valladolid, IMUVa, Spain)

ABSTRACT

Let C be a curve with only one place at infinity, and let $S_{C,\infty}$ be its semigroup at infinity. It is known that this semigroup is generated by a δ-sequence in the set of the positive integer numbers. In this work we study the families of δ-sequences which generate the same semigroup $S_{C,\infty}$. We also introduce the notion of *minimal* δ-sequences as those with least length among all the δ-sequences generating the same semigroup at infinity. Finally, we prove that the arithmetic genera of the strict transforms of all the curves by their embedded resolutions whose δ-sequence determines $S_{C,\infty}$ coincide and are determined by the conductor of $S_{C,\infty}$.

Poster: "*Equiaffine structure on frontals*"

by Igor Chagas Santos (ICMC-USP, Brazil)

ABSTRACT

In this poster we generalize the idea of equiaffine structure to the case of frontals, in order to define the Blaschke vector field of a frontal. We also investigate the conditions that a frontal needs to satisfy in order to have a Blaschke vector field and provide some examples. Finally, as an application of this theory, we present a version of the fundamental theorem of affine differential geometry for frontals.

Poster Session II

Poster: "*The Euler Obstruction of a Map on an Isolated Determinantal Singularity*"

by Raphael de Omena Marinho (ICMC-USP, Brazil)

ABSTRACT

An important direction of investigation in Singularity Theory is the search for local invariants associated with singular varieties. A central invariant is the Milnor number defined for hypersurfaces with isolated singularity and isolated complete intersection singularities. A natural step to advance this investigation is to consider Determinantal Singularities, since they appear as a natural generalization of complete intersections.
In this case, we have other invariants that can be seen as generalizations of the Milnor number. One of them is the Euler obstruction of a function with an isolated critical point at the origin, which has been studied by many authors.
The Euler obstruction of a map generalizes the construction of the Euler obstruction of a function, and is expressed in terms of the number of critical points in an appropriate Morsification of the coordinate functions of the map. This statement is presented by Grulha, Ruiz e Santana **[1]**.

Our goal is relating the Euler obstruction of a map with other invariants, such as the vanishing Euler characteristic, the polar multiplicity and the local Euler obstruction, on an isolated determinantal singularity. We also compute them for simple isolated Cohen-Macaulay codimension 2 singularities. This a joint work with Nivaldo Grulha Jr., and Miriam Pereia.

[1] N. G. Grulha Jr, C. M. Ruiz, H. Santana, The geometrical information encoded by the Euler obstruction of a map, *International Journal of Mathematics, 33(04):2250029, 2022.*

Poster: "*Existence of OPSUs*"

by Nacho Breva (Universitat de València, Spain)

ABSTRACT

On this poster we explain how to refine Mather's method to obtain stable unfoldings, allowing us to get the one with the minimal number of parameters. We also give some results to detect when a map-germ admits a 1-parameter stable unfolding (OPSU). We show that, with finite A_e-codimension, these map-germs admit a versal unfolding with a nice form, and give some interesting examples.

Poster Session II

Poster: "*Analytic semiroots for plane branches with one Puiseux pair*"

by David Senovilla Sanz (Universidad de Cantabria, Spain)

ABSTRACT

The analytic class of a germ of a plane curve C is partially determined by a set of discrete values called ‘’semimodule of differential values’’, this set can be obtained via some representative set of 1-forms in two variables, which will be a minimal standard basis of the module of differentials of C. These 1-forms have a 1-parameter family of invariant curves with the same topological type as the original curve. These curves are the analytic semiroots of C. We can relate the semimodule of differential values of the analytic semiroots with the one of C: they are obtained by a truncation of the list of generators of the initial semimodule at the corresponding differential value.

Poster: "*Milnor Fiber Consistency via Flatness*"

by Alex Hof (University of Wisconsin-Madison, USA)

ABSTRACT

The Milnor fiber of a holomorphic function defining an isolated singularity can be understood by perturbing the function slightly to one with only Morse critical points. For an arbitrary-dimensional singularity, perturbation is no longer guaranteed to preserve the Milnor fiber, making an analogous approach difficult. We present a new algebraic condition - the flatness of the critical locus over the parameter space - under which this problem does not arise, giving a new avenue for Milnor fiber computations.