Workshop MEGAR – Effective Methods in Real Algebraic Geometry
Madrid, June 21–22, 2019

Quadrature Rules and their Geometry

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A quadrature rule of a measure μ on the real line $\mathbb{R}$ or the real plane $\mathbb{R}^2$ consists in a convex combination of finitely many evaluations at points, called nodes, that agrees with integration against μ for all polynomials up to some fixed degree d. In this talk we will use the duality of positive polynomials and moments to analyse the geometry of Quadrature rules. We will show how optimisation approaches can be used to bound the number of nodes necessary. This yields new bounds in particular in the case of the plane. On the real line, a quadrature rule with the minimal number of nodes is called Gaussian quadrature. Such a quadrature is unique for degree $2d-1$. For even degrees we will construct a bivariate polynomial whose roots parametrize the nodes of minimal quadrature rules for measures on the real line. We give two symmetric determinantal formulas for this polynomial, which translate the problem of finding the nodes to solving a generalized eigenvalue problem.

(based on joint works with G. Blekherman, M. Kummer, M. Schweighofer, and C. Vinzant)

Keywords: Quadrature rules, positive polynomials, optimisation

References

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[2] C. Riener, M. Schweighofer, *Optimization approaches to quadrature: new characterizations of Gaussian quadrature on the line and quadrature with few nodes on plane algebraic curves, on the plane and in higher dimensions*. *Journal of Complexity* **45**(2), 22–54 (2018).

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