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Moderately Discontinuous Algebraic Topology: a new language to study metric degenerations

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In the works [1] and [2] we develop a new metric algebraic topology, called the Moderately Discontinuous Homology and Homotopy, in the context of a germ of a topological cone in \mathbb{R}^n and more generally of a (degenerating) continuous family of sets in $\mathbb{R}^n \times \mathbb{R}$. This set or family is endowed with a nice metric structure such as the restriction of the euclidean metric in the ambient space, or the continuous extension of the riemannian metric in the smooth part. Then, our theory captures bilipschitz information or in other words, quasi isometric invariants, and aims to codify part of the bilipschitz geometry.

For example, in the case of a topological cone (which in general is not metrically a straight cone), the moderately discontinuous theory captures the different speeds, with respect to the distance to the origin of the cone, in which the topology of the link collapses towards the origin. Similarly, in a degenerating family, it captures the different speeds of collapsing with respect to the family parameter.

In this talk, I will give a gentle introduction to the theory, and explain the context in which it is originally established, but that could be enlarged in order to be applied in a more softer context.

References

[1] (with J. Fernández de Bobadilla, S. Heinze, E. Sampaio) *Moderately discontinuous homology*. Comm. Pure App. Math. https://doi.org/10.1002/cpa.22013. Also available in arXiv:1910.12552v3.

[2] (with J. Fernández de Bobadilla, S. Heinze) *Moderately discontinuous homotopy*. IMRN. Available in ArXiv:2007.01538.