

How many real eigenvalues do Bohemian Matrices have?

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Bohemian matrices are families of matrices whose entries come from a fixed discrete set of small integers. The term is a contraction of BOUNDED HEIGHT MATRIX OF INTEGERS and was coined by R. Corless and S. Thornton. Extensive computations of eigenvalues and characteristic polynomials of Bohemian matrices have led to interesting observations and conjectures (see <http://www.bohemianmatrices.com> and [1]). We will analyse two different problems regarding the number of different real eigenvalues for Bohemian matrices when their size is fixed. The first one asks for the most frequent number of different real eigenvalues for an Upper Hessenberg and Toeplitz matrix of size n with 1 in the first subdiagonal and when the other entries come from the set $\{-1, 0, 1\}$ (for n fixed, there are such 3^n matrices). We will show how an extensive computation provides one (unproven yet) conjecture for the previously introduced question. The second one is related with the Bohemian Correlation Matrices. A symmetric matrix is a correlation matrix if it has ones on the diagonal and its eigenvalues are nonnegative. We will show that the number of Bohemian Correlation Matrices over 0 and 1 are linked with Bell numbers (OEIS A000110 - <http://oeis.org/A000110>) and will introduce a new characterisation of correlation matrices in order to analyse what happens with these numbers when the entries come from the set $\{-1, 0, 1\}$ and to solve some correlation matrix completion problems arising in risk management and insurance (see [2]).

Keywords: Bohemian matrices, Real eigenvalues, Correlation matrices

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