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Methods  
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Métodos

**M E A G**

in Algebraic  
in Algebraischer  
в Алгебраической  
Algebraica  
Algébrique  
Algebraica

Geometry  
Geometrie  
Геометрии  
in Geometria  
en Géométrie  
en Geometría

## **MEGA 2019**

Madrid, June 17-21

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# MEGA 2019: Effective Methods in Algebraic Geometry Madrid, June 17–21 2019

## FOREWORD

The conference MEGA 2019 will take place at Universidad Complutense de Madrid (Madrid, Spain) in June 17-21, 2019.

MEGA is the acronym for Effective Methods in Algebraic Geometry (and its equivalent in Italian, French, Spanish, German, Russian, etc.). This series of biennial international conferences, with the tradition dating back to 1990, is devoted to computational and application aspects of Algebraic Geometry and related topics, over any characteristics.

MEGA conferences have emerged from the leading research activity in Effective Algebraic Geometry developed in Europe over the last decades. They have established themselves as an important vehicle in the exploration of the broad interactions between Algebraic Geometry and Computational and Applied domains of Mathematics. The conferences have also increasingly been attracting young talented researchers, showing the strength and vitality of the community. The MEGA conferences aim at promoting the development of the subject through academic exchange between International researchers; at presenting the recent progresses of the domain through the presentation of selected talks; at updating the community knowledge through the invitation of talented researchers from Europe and outside; and at attracting and supporting young students and researchers to join the current research area.

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- Jaroslaw Wisniewski (Warsaw, Poland)



PLENARY TALKS

## Algorithms for Bohemian Matrices

20J  
9:30  
S118

Robert M. Corless<sup>1</sup>

A "Bohemian" matrix family is a set of matrices with elements drawn from the same population—typically a fixed discrete set, usually integers, and often just the numbers  $\{-1, 0, 1\}$ . The mnemonic comes from Bounded HEight Matrix of Integers (BOHEMI). These objects are studied in many places, but not until recently under that name: they occur in graph theory (of course), compressive sensing, optimization, mathematical biology, and many more areas. My own original interest was sparked (at EACA 2004 in Santander) by simple software testing, and indeed by using these we have uncovered bugs in several major software packages. But there are some interesting open mathematical questions that have arisen when considering Bohemian matrices as objects of study in their own right, including combinatorial conjectures now listed in the Online Encyclopedia of Integer Sequences (oeis.org). In this talk I will discuss some algorithmic changes that arise on contemplating "brute force" attacks on Bohemian Eigenvalue problems: the typical numerical algorithms that are so valuable for computing eigenvalues of a few large matrices become noticeably less attractive when the problem is instead to compute or count distinct eigenvalues of very many (indeed exascale many) small matrices. This leads to the Characteristic Polynomial Database and its uses. This is joint work with Steven E. Thornton and the other members of the Bohemian Eigenvalue Project, including Eunice Y. S. Chan, Laureano Gonzalez-Vega, J. Rafael Sendra, and Juana Sendra.

<sup>1</sup>University of Western Ontario

## Geometry of nonnegative rank

18J  
9:30  
S118

Kubjas Kaie<sup>1</sup>

One of many definitions gives the rank of an  $m \times n$  matrix  $M$  as the smallest natural number  $r$  such that  $M$  can be factorized as  $AB$ , where  $A$  and  $B$  are  $m \times r$  and  $r \times n$  matrices respectively. In many applications, we are interested in factorizations of a particular form. For example, factorizations with nonnegative entries define the nonnegative rank which is a notion that is used in data mining applications, statistics, complexity theory etc. Nonnegative rank has geometric characterizations using nested polytopes. I will explain how to use these characterizations to study the semialgebraic geometry of the set of matrices of given nonnegative rank. In particular, I will recall what was previously known about the set of matrices of nonnegative rank at most  $r$  for  $r = 1, 2, 3$  and present recent results on the boundaries of the set of matrices of nonnegative rank at most four using notions from the rigidity theory of frameworks. These results are based on joint work with Robert Krone.

<sup>1</sup>Aalto University

# Post-quantum Cryptography with high degree polynomials

Ignacio Luengo<sup>1</sup>

Post-quantum cryptography is the public-key cryptography resistant to future quantum computers. In this talk we will talk about a post-quantum cryptosystem called DME (Double Matrix Exponentiation) based on on high degree polynomials on a small number of variables that I have developed (using ideas of Algebraic Geometry), patented and present it to the NIST contest to choose the future post-quantum cryptography standard. I will also present some Commutative Algebra open questions related with the algebraic cryptoanalysis of the scheme DME.

<sup>1</sup>Universidad Complutense de Madrid

21J  
9:30  
S118

# Algebraic Phylogenetics

Mateusz Michałek<sup>1</sup>

Phylogenetics is a science that aims at reconstructing the history of evolution. Phylogenetic tree models are generalizations of well-known Markov chains. In my talk I will present so-called group-based models and their relations to algebra and combinatorics. To a model of evolution one associates an algebraic variety that is the Zariski closure of points corresponding to probability distributions allowed by the model. Many important varieties arise by this construction, e.g. secant varieties of Segre products of projective spaces. It turns out that group-based models provide toric varieties. In particular, they may be studied using tools from toric geometry relating to combinatorics of lattice polytopes.

<sup>1</sup>Max Planck Institute

19J  
12:20  
S118

# The roots of solving polynomial equations

Bernard Mourrain<sup>1</sup>

Solving polynomial equations is a key ingredient of many Effective Methods in Algebraic Geometry. We will revisit this problem from its roots for zerodimensional systems, developing its links with normal form computation, Grobner basis, border basis, providing explicit descriptions of the Hilbert scheme of points and exploiting these properties in moment problems.

<sup>1</sup>Inria Sophia Antipolis Méditerranée

17J  
10:00  
S118

# Classification of lattice polytopes

Francisco Santos<sup>1</sup>

I will report on some recent projects on the classification of lattice polytopes with various properties. Most notably, together with O. Iglesias-Valiño we have recently completed the classification of "empty 4-simplices", a task started by Mori, Morrison and Morrison almost exactly 30 years ago. An empty simplex is a lattice simplex with no lattice point other than its vertices. "Classification" means modulo affine unimodular transformations of the ambient

18J  
15:00  
S118

lattice. The classification of 3-dimensional empty simplices was completed by G. White in 1964; there are infinitely many of them, but they can all be described as one family depending on two integer parameters. In dimension four the classification is much wilder: there is one 3-parameter family analogous to the one in dimension three, but also two 2-parameter families, fifty-two 1-parameter families, and 2461 sporadic examples. Interest in empty simplices comes (among other places) from the minimal model program in algebraic geometry, where these simplices correspond almost bijectively to rational quotient singularities.

<sup>1</sup>Universidad de Cantabria

## **Bounding Betti numbers of real hypersurfaces near the tropical limit**

19J  
9:30  
S118

**Kristin Shaw**<sup>1</sup>

Almost 150 years ago Harnack proved a tight upper bound on the number of connected components of a real planar algebraic curve of degree  $d$ . However, in higher dimensions we know very little about the topology of real algebraic hypersurfaces. For example, we do not know the maximal number of connected components of real quintic surfaces in projective space. In this talk I will explain the proof of a conjecture of Itenberg which, for a particular class of real algebraic projective hypersurfaces, bounds all Betti numbers, not only the number of connected components, in terms of the Hodge numbers of the complexification. The real hypersurfaces we consider arise from Viro's patchworking construction, which is an effective and combinatorial method for constructing topological types of real algebraic varieties. To prove the bounds conjectured by Itenberg we develop a real analogue of tropical homology and use spectral sequences to compare it to the usual tropical homology of Itenberg, Katzarkov, Mikhalkin, Zharkov. Their homology theory gives the Hodge numbers of a complex projective variety from its tropicalization. Lurking in the spectral sequence of the proof are the keys to having combinatorial control of the topology of the real hypersurface produced from a patchwork in any toric variety. This is joint work with Arthur Renaudineau.

<sup>1</sup>University of Oslo

## **Completely log-concave polynomials and matroids**

17J  
15:00  
S118

**Cynthia Vinzant**<sup>1</sup>

Stability is a multivariate generalization for real-rootedness in univariate polynomials. Within the past ten years, the theory of stable polynomials has contributed to breakthroughs in combinatorics, convex optimization, and operator theory. I will introduce a generalization of stability, called complete log-concavity, that satisfies many of the same desirable properties. These polynomials were inspired by work of Adiprasito, Huh, and Katz on combinatorial Hodge theory, but can be defined and understood in elementary terms. The structure of these



polynomials is closely tied with notions of discrete convexity, including matroids, submodular functions, and generalized permutohedra. I will discuss the beautiful real and combinatorial geometry underlying these polynomials and applications to problems in matroid theory. This is based on joint work with Nima Anari, Kuikui Liu, and Shayan Oveis Gharan.

<sup>1</sup>North Carolina State University



CONTRIBUTED TALKS

## Autocovariance varieties of moving average random fields

17J  
16:10  
B16

Carlos Amendola<sup>1</sup>, Viet Son Pham<sup>1</sup>

We study an interesting family of varieties coming from a standard stochastic model in time series analysis, namely moving average random fields. The problem of parameter estimation for this model via the autocovariance function can be linked to the algebraic invariants of euclidean distance degree and maximum likelihood degree. We illustrate this connection with concrete examples and in our computations we use tools from commutative algebra and numerical algebraic geometry.

<sup>1</sup>Technical University of Munich

## Classification of triples of lattice polytopes with a given mixed volume in dimension three

18J  
12:30  
B16

Gennadiy Averkov<sup>1</sup>, Christopher Berger<sup>1</sup>, Ivan Soprunov<sup>2</sup>

Given a system of  $d$  Laurent polynomials  $f_1, \dots, f_d$  in  $d$  variables, a natural invariant to study is the tuple of corresponding Newton polytopes  $(N(f_1), \dots, N(f_d))$ . For example, the classical Bernstein–Khovanskii–Kouchnirenko theorem states that the number of common roots in the complex torus of a generic system  $f_1, \dots, f_d$  can be computed as the so-called mixed volume  $MV(N(f_1), \dots, N(f_d))$  of the Newton polytopes.

We combine computational methods from discrete geometry with results from Brunn–Minkowski theory in order to completely classify inclusion-maximal triples of mixed volume at most 4. By the above correspondence, this produces a classification of general trivariate sparse polynomial systems with up to 4 solutions in the complex torus.

<sup>1</sup>Otto-von-Guericke Universität Magdeburg

<sup>2</sup>Cleveland State University

## Vandermonde varieties, mirror spaces, and the cohomology of symmetric semi-algebraic sets

18J  
16:10  
B16

Saugata Basu<sup>1</sup>, Cordian Riener<sup>2</sup>

Let  $d, k \in \mathbb{N}$ ,  $3 < d \leq k$ ,  $\mathbf{y} = (y_1, \dots, y_d) \in R^d$ , and let  $V_{d,\mathbf{y}}^{(k)}$  denote the Vandermonde variety defined by  $p_1^{(k)} = y_1, \dots, p_d^{(k)} = y_d$ , where  $p_j^{(k)} = \sum_{i=1}^k X_i^j$ . Then, the cohomology groups  $H^*(V_{d,\mathbf{y}}^{(k)}, \mathbb{Q})$  have the structure of  $\mathfrak{S}_k$ -modules. We prove that for all  $\lambda \vdash k$ , the multiplicity of the Specht-module  $\mathbb{S}^\lambda$  in  $H^i(V_{d,\mathbf{y}}^{(k)}, \mathbb{Q})$  is zero if  $\text{length}(\lambda) \geq i + 2d - 3$ . This

vanishing result allows us to prove a similar vanishing result for arbitrary symmetric semi-algebraic sets defined by symmetric polynomials of degrees bounded by  $d$ . As a result, we obtain for each fixed  $\ell \geq 0$ , an algorithm for computing the first  $\ell + 1$  Betti numbers of such sets, whose complexity is polynomially bounded (for fixed  $d$  and  $\ell$ ).

<sup>1</sup>Purdue University

<sup>2</sup>The Arctic University of Norway

## An Algorithmic Approach to Classify Quartic Monoid Surfaces of $\mathbb{P}_K^3$

**Mauro Carlo Beltrametti<sup>1</sup>, Alessandro Logar<sup>2</sup>, Maria Laura Torrente<sup>1</sup>**

Quartic surfaces of the projective space are a classical subject of algebraic geometry and their study goes back for a long time now. In more recent years, several authors have studied the class of monoid quartic surfaces, an important subclass of quartic surfaces, at least because these are rational surfaces, with the aim of describing the geometry and giving a classification of their possible singularities. In this paper we address the problem of classifying all the possible configurations of lines contained in a quartic monoid surface in  $\mathbb{P}_K^3$ . Since there are several thousands of cases to consider, each of them with some peculiarities, it is quite necessary to develop suitable algorithms to get the desired answers. Most of the work has been therefore dedicated to develop specific software to automatise the classification in order to contribute in proving our statements. As a final result we get a description of all the possible configuration of lines that can be contained in a quartic monoid surface. In order to obtain our results, we have intensively used the computer algebra systems CoCoA and Sage.

<sup>1</sup>Università di Genova

<sup>2</sup>Università degli Studi di Trieste

21J  
11:00  
S118

## Waring, tangential and cactus decompositions

**Alessandra Bernardi<sup>1</sup>, Daniele Taufer<sup>1</sup>**

We review the famous algorithm of Brachat, Comon, Mourrain and Tsingaridas for symmetric tensor decomposition. Afterwards, we generalize it in order to detect possibly different decompositions involving points on the tangential variety of a Veronese variety. Finally, we produce an algorithm for cactus rank and decomposition, which also detects the support of the minimal apolar scheme and its length at each component.

<sup>1</sup>University of Trento

18J  
11:50  
B15

## The monic rank and instances of Shapiro's Conjecture

**Arthur Bik<sup>1</sup>, Jan Draisma<sup>1</sup>, Alessandro Oneto<sup>2</sup>, Emanuele Ventura<sup>3</sup>**

A Conjecture by Shapiro states that every homogeneous polynomial in  $\mathbb{C}[x, y]$  of degree  $de$  is the sum of at most  $d$   $d$ -th powers of homogeneous polynomials of degree  $e$ . In this talk, we prove a few instances of this conjecture. We do this by introducing the monic rank. For a

20J  
11:00  
B16

polynomial, the monic rank is the minimal  $k$  for which we can write (an appropriately scaled version of) the polynomial as a sum of  $k$   $d$ -th powers of monic polynomials. We prove that the maximal monic rank of polynomials of degree  $de$  is at most  $d$  when  $e = 1$ ,  $d \in \{1, 2\}$  or  $(d, e) \in \{(3, 2), (3, 3), (3, 4), (4, 2)\}$ .

<sup>1</sup>University of Bern

<sup>2</sup>Barcelona Graduate School of Mathematics and Universitat Politècnica de Catalunya

<sup>3</sup>Texas A & M University

## The lexicographic degree of the first two-bridge knots

19J  
11:00  
B16

**Erwan Brugallé<sup>1</sup>, Pierre-Vincent Koseleff<sup>2</sup>, Daniel Pecker<sup>2</sup>**

We study the degree of polynomial representations of knots. We give the lexicographic degree of all two-bridge knots with 11 crossings or fewer. First, we estimate the total degree of a lexicographic parametrisation of such a knot. This allows us to transform this problem into a study of real algebraic trigonal plane curves, and in particular to use the braid theoretical method developed by Orevkov.

We describe our algorithm: first to obtain explicit constructions of trigonal curves corresponding to *simple* diagrams, second to prove the minimality of their degrees by computing the associated braids and testing their quasipositivity.

<sup>1</sup>Université de Nantes, Laboratoire de Mathématiques Jean Leray (CNRS LMJL UMR 6629), France

<sup>2</sup>Sorbonne Université (UPMC–Paris 6), Institut de Mathématiques de Jussieu (CNRS IMJ-PRG UMR 7586) & Inria-Paris, France

<sup>3</sup>Sorbonne Université (UPMC–Paris 6), Institut de Mathématiques de Jussieu (CNRS IMJ-PRG UMR 7586), France

## Generalized weight properties of resultants and discriminants, and applications to projective enumerative geometry

17J  
17:10  
B15

**Laurent Busé<sup>1</sup>, Thomas Dedieu<sup>2</sup>**

In a book dating back to 1862, Salmon stated a formula giving the first terms of the Taylor expansion of the discriminant of a plane algebraic curve, and from it derived various enumerative quantities for surfaces in  $\mathbf{P}^3$ . In this text, we provide complete proofs of this formula and its enumerative applications, and extend Salmon’s considerations to hypersurfaces in a projective space of arbitrary dimension. To this end, we extend reduced elimination theory by introducing the concept of reduced discriminant, and provide a thorough study of its weight properties; the latter are deeply linked to projective enumerative geometric properties. Then, following Salmon’s approach, we compute the number of members of a pencil of hyperplanes that are bitangent to a fixed projective hypersurface. Some other results in the same spirit are also discussed.

<sup>1</sup>Université Côte d’Azur

<sup>2</sup>Université de Toulouse

## The Rees algebra of parametric curves via liftings

**Teresa Cortadellas Benitez<sup>1</sup>, David A. Cox<sup>2</sup>, Carlos D’Andrea<sup>1</sup>**

17J

18:30

B15

We study the defining equations of the Rees algebra of ideals arising from curve parametrizations in the plane and in rational normal scrolls, inspired by the work of Madsen and Kustin, Polini and Ulrich. The curves are related by work of Bernardi, Gimigliano and Idá, and we use this framework to relate the defining equations.

<sup>1</sup>Universitat de Barcelona

<sup>2</sup>Amherst College

## Uniform matrix product states from an algebraic geometer’s point of view

**Adam Czapliński<sup>1</sup>, Mateusz Michałek<sup>2</sup>, Tim Seynnaeve<sup>3</sup>**

17J

17:10

B16

We apply methods from algebraic geometry to study uniform matrix product states. Our main results concern the topology of the locus of tensors expressed as uMPS, their defining equations and identifiability. By an interplay of theorems from algebra, geometry and quantum physics we answer several questions and conjectures posed by Critch, Morton and Hackbusch.

<sup>1</sup>Universität Siegen

<sup>2</sup>Max Planck Institute for Mathematics in the Sciences, Leipzig, Polish Academy of Sciences, and Aalto University

<sup>3</sup>Max Planck Institute for Mathematics in the Sciences, Leipzig

## Regular cylindrical algebraic decomposition

**James Davenport<sup>1</sup>, Acyr Locatelli<sup>1</sup>, Gregory Sankaran<sup>1</sup>**

18J

11:50

B16

Various algorithms exist to construct cylindrical algebraic decompositions of semi-algebraic sets in  $\mathbb{R}^n$ , and thus to represent such a set  $S$  as a cell complex. However, this is not enough to allow computation of topological invariants of  $S$ . Such computations could be carried out by representing  $S$  as a simplicial complex, but that typically involves so much subdivision as to be computationally infeasible. Instead, we seek to represent  $S$  as a regular cell complex: this is a cell complex with a simple extra condition on the boundary of each cell, and allows computation in the same way as in a simplicial complex.

We show that a strong well-based cylindrical algebraic decomposition  $\mathcal{P}$  of a bounded semi-algebraic set  $S$  is a regular cell decomposition, in any dimension and independently of the method by which  $\mathcal{P}$  is constructed. Being well-based is a global condition on  $\mathcal{P}$  that holds for the output of many widely used algorithms. We also show the same for  $S$  of

dimension  $\leq 3$  and  $\mathcal{P}$  a strong cylindrical algebraic decomposition that is locally boundary simply connected: this is a purely local extra condition.

<sup>1</sup>Department of Mathematical Sciences, University of Bath, UK

## On Catalan-many tropical morphisms to trees

18J  
17:50  
B15

**Jan Draisma<sup>1</sup>, Alejandro Vargas<sup>1</sup>**

We recall the classical notion of curve gonality and a dictionary to go from classical to tropical geometry, to then investigate the tree gonality of a metric graph  $\Gamma$ , defined as the minimum degree of a tropical morphism from any tropical modification of  $\Gamma$  to  $\Delta$ , with  $\Delta$  any metric tree. We give a combinatorial constructive proof that this number is at most  $d = \lceil g/2 \rceil + 1$ , where  $g$  is the genus of  $\Gamma$ . For even  $g$  we construct a space parametrizing these tropical morphisms, with properties that also match a classical space. Thus, for all genera we have a case by case algorithm that given  $\Gamma$ , finds catalan many tropical morphisms with degree  $d$  from (a tropical modification of)  $\Gamma$  to a metric tree, and for even genus the list exhausts all possibilities.

<sup>1</sup>Universität Bern

## The dual cone of sums of non-negative circuit polynomials

20J  
11:40  
B16

**Mareike Dressler<sup>1</sup>, Helen Naumann<sup>2</sup>, Thorsten Theobald<sup>2</sup>**

For the set  $\mathcal{A}$  of integer points of a convex lattice polytope in  $\mathbb{R}_+^n$ , denote by  $C_{\text{sonc}}(\mathcal{A}) \in \mathbb{R}[x_1, \dots, x_n]$  the cone of sums of non-negative circuit polynomials with support  $\mathcal{A}$ . We derive a representation of the dual cone  $(C_{\text{sonc}}(\mathcal{A}))^*$  and deduce a resulting optimality criterion for the use of sums of non-negative circuit polynomials in polynomial optimization.

<sup>1</sup>Institute for Computational and Experimental Research in Mathematics (ICERM)

<sup>2</sup>Goethe Universität

## Certification for Polynomial Systems via Square Subsystems

21J  
11:00  
B16

**Timothy Duff<sup>1</sup>, Frank Sottile<sup>2</sup>**

We consider numerical certification of approximate solutions to an overde-termined system of  $N$  polynomial equations in  $n$  variables where  $n < N$  by passing to a square subsystem. We give two approaches which rely upon additional intersection-theoretic information. The excess solutions to a square subsystem are counted by a birationally-invariant intersection index or Newton-Okounkov body. When this number is known, we explain how to certify individual solutions to the original overdetermined system. When the number of solutions to both systems are known, we explain how to certify all solutions to the overdetermined system.

<sup>1</sup>Georgia Institute of Technology

<sup>2</sup>Texas A&M University

# Computing minimal Gorenstein covers

Joan Elias<sup>1</sup>, Roser Homs<sup>1</sup>, Bernard Mourrain<sup>2</sup>

We analyze and present an effective solution to the minimal Gorenstein cover problem: given a local Artin  $\mathbf{k}$ -algebra  $A = \mathbf{k}[[x_1, \dots, x_n]]/I$ , compute an Artin Gorenstein  $\mathbf{k}$ -algebra  $G = \mathbf{k}[[x_1, \dots, x_n]]/J$  such that  $\ell(G) - \ell(A)$  is minimal. We approach the problem by using Macaulay's inverse systems and a modification of the integration method for inverse systems to compute Gorenstein covers. We propose new characterizations of the minimal Gorenstein cover and present a new algorithm for the effective computation of the variety of all minimal Gorenstein covers of  $A$  for low Gorenstein colength. Experimentation illustrates the practical behavior of the method.

<sup>1</sup>Universitat de Barcelona, Spain

<sup>2</sup>INRIA - Sophia Antipolis Méditerranée, France

17J  
11:30  
B15

# On the number of real zeros of random fewnomials

Alperen Ergür<sup>1</sup>, Peter Bürgisser<sup>1</sup>, Josue Tonelli-Cueto<sup>1</sup>

If a univariate polynomial has  $t$  nonzero terms then it has at most  $(t - 1)$  many positive real zeros said René Descartes in his *La Géométrie* around 1637. Since then it's an intriguing puzzle to find the correct multivariate generalization of Descartes' rule. Around 1980, Askold G. Khovanskii showed that a system of real polynomials with  $n$  variables and  $t$  nonzero terms has at most exponentially many non-degenerate positive real zeros in terms of  $t$ . A conjecture attributed to Anatoli Kushnirenko claims that the correct upper bound is of order  $t^n$ . This remains open even for  $n = 2$ ! We show that Kushnirenko's prediction holds true for random sparse polynomial systems with a fixed support set and Gaussian coefficients of arbitrary variance. This is joint work with Peter Bürgisser and Josue Tonelli-Cueto.

<sup>1</sup>Technische Universität Berlin

20J  
11:00  
B15

# A characterization of graphs admitting flexible realizations on the sphere

Matteo Gallet<sup>1</sup>, Georg Grasegger<sup>2</sup>, Josef Schicho<sup>3</sup>

We interpret realizations of a graph on the sphere up to rotations as elements of a moduli space of curves of genus zero. This allows us to provide a combinatorial characterization of graphs that admit a flexible assignment of edge lengths on the sphere, in terms of the existence of particular colorings of the edges. Moreover, we determine relations between the spherical lengths of the edges that are necessary for flexibility.

<sup>1</sup>International School for Advanced Studies, Trieste, Italy

<sup>2</sup>Research Institute for Computational and Applied Mathematics, Linz, Austria

<sup>3</sup>Johannes Kepler University, Linz, Austria

17J  
11:30  
B16

## The rough Veronese variety

18J  
12:30  
B15

**Francesco Galuppi**

We study signature tensors of paths from an algebraic geometric viewpoint. The signatures of a given class of paths parametrize a variety inside the space of tensors. These "signature varieties" provide both new tools to investigate paths and new challenging questions about their behavior. In this talk I will focus on the class of rough paths. They play a central role in stochastic analysis, and their signature variety has peculiar geometric properties, showing surprising analogies with the classical Veronese variety. We show that this so-called Rough Veronese is toric. This makes it much easier to study it and to perform explicit computations.

<sup>1</sup>Max Planck Institute for Mathematics in the Sciences, Leipzig

## Approximate super-resolution and truncated moment problems in all dimensions

20J  
11:40  
B15

**Hernán García<sup>1</sup>, Camilo Hernández<sup>2</sup>, Mauricio Junca<sup>1</sup>, Mauricio Velasco<sup>1</sup>,**

We study the problem of reconstructing a discrete measure on a compact set  $K \subset \mathbb{R}^n$  from a finite set of moments (possibly known only approximately) via convex optimization. We give new uniqueness results, new quantitative estimates for approximate recovery and a new sum-of-squares based hierarchy for approximate super-resolution on compact semi-algebraic sets.

<sup>1</sup>Universidad de los Andes

<sup>2</sup>Columbia University

## Stronger bounds on the cost of computing Groebner bases for HFE systems

17J  
17:50  
B15

**Elisa Gorla<sup>1</sup>, Daniella Mueller<sup>2</sup>, Christoph Petit<sup>3</sup>**

We give upper bounds for the solving degree and the last fall degree of the polynomial system associated to the HFE (Hidden Field Equations) cryptosystem. Our bounds improve the known bounds for this type of systems.

<sup>1</sup>Université de Neuchâtel

<sup>2</sup>University College Dublin

<sup>3</sup>University of Birmingham

## Hurwitz numbers and piecewise polynomiality properties

19J  
11:40  
B15

**Marvin Hahn<sup>1</sup>**



Hurwitz numbers count branched genus  $g$ , degree  $d$  coverings of the Riemann sphere with fixed ramification data. These enumerative objects are deeply intertwined with Gromov-Witten theory. In recent years several related notions of Hurwitz-type counts and combinatorial interpolations between them appeared in the literature and the uncovering of the connections between these counts and Gromov Witten theory – similar to the classical case – has become an active field of research. One of the crucial necessary properties for such connections to exist is a (piecewise) polynomial behaviour of the Hurwitz-type counts in the initial ramification data. This talk is centered around the polynomial structure and wall-crossing behaviour of interpolations between double, monotone double and Grothendieck dessin d'enfants double Hurwitz numbers. Parts of this talk are based on joint work with Reinier Kramer and Danilo Lewanski.

<sup>1</sup>Goethe Universität Frankfurt am Main

## Attainable Regions of Dynamical Systems

Nidhi Kaihnsa<sup>1</sup>

17J  
17:50  
B16

We present a mathematical definition for the attainable region of a dynamical system, with primary focus on mass action kinetics for chemical reactions. We characterise this region for linear dynamical systems, and we report on experiments on weakly reversible systems with one linkage class. A construction due to Vinzant is adapted to give a representation of faces in the convex hull of trajectories.

<sup>1</sup>Max Planck Institute for Mathematics in the Sciences, Leipzig

## A decomposition of The Hilbert Scheme given by Gröbner schemes

Yuta Kambe<sup>1</sup>

18J  
17:10  
B15

We consider the Hilbert scheme parameterizing closed subschemes of the projective  $n$ -space with fixed Hilbert polynomial. If we think to the set of  $k$ -rational points of the Hilbert scheme as represented by a set of homogeneous ideals in  $S = k[x_0, \dots, x_n]$ , then  $k$ -rational points can be decomposed into loci of homogeneous ideals with fixed initial ideal, called Gröbner strata or Gröbner schemes. The main purpose of this talk is to discuss this decomposition and its relation with a torus group action on the Hilbert scheme.

<sup>1</sup>Saitama University

## Complete Complexes and Spectral Sequences

Mikhail Kapranov<sup>1</sup>, Evangelos Routis<sup>1</sup>

19J  
11:00  
B15

The space of complete collineations is an important and beautiful chapter of algebraic geometry, which has its origins in the classical works of Chasles, Giambeli, Hirst, Schubert, Tyrell and others, dating back to the 19th century. It provides a ‘wonderful compactification’ (i.e. smooth with normal crossings boundary) of the space of full-rank maps between two (fixed) vector spaces. More recently, the space of complete collineations has been studied

intensively and has been used to derive groundbreaking results in diverse areas of mathematics. One such striking example is L. Lafforgue's compactification of the stack of Drinfeld's shtukas, which he subsequently used to prove the Langlands correspondence for the general linear group.

We look at these classical spaces from a modern perspective: a complete collineation is simply a spectral sequence of two-term complexes of vector spaces. We then develop a theory involving more full-fledged (simply graded) spectral sequences of complexes of vector bundles with arbitrarily many terms. We prove that the set of such spectral sequences has the structure of a smooth projective variety, the 'variety of complete complexes', which provides a desingularization, with normal crossings boundary, of the 'Buchsbaum-Eisenbud variety of complexes', i.e. a 'wonderful compactification' of the union of its maximal strata.

<sup>1</sup>The University of Tokyo

## Computing Quotients by Connected Solvable Groups

Gregor Kemper<sup>1</sup>

17J  
12:10  
B15

Consider an action of a connected solvable group  $G$  on an affine variety  $X$ . This talk presents an algorithm that constructs a semi-invariant  $f \in K[X] =: R$  and computes the invariant ring  $(R_f)^G$  together with a presentation. The morphism  $X_f \rightarrow \text{Spec}((R_f)^G)$  obtained from the algorithm is a universal geometric quotient. In fact, it is even better than that: a so-called excellent quotient. If  $R$  is a polynomial ring, the algorithm requires no Gröbner basis computations. If  $R$  is a complete intersection, then so is  $(R_f)^G$ .

<sup>1</sup>Technische Universität München

## Computing representation matrices for the Frobenius on cohomology groups

Momonari Kudo<sup>1</sup>

17J  
18:30  
B16

In this talk, we present an algorithm to compute representation matrices for the Frobenius map on the cohomology groups of general algebraic varieties over a perfect field of positive characteristic. We also propose an efficient algorithm specific to complete intersections. Our algorithms shall derive fruitful applications such as computing Hasse-Witt matrices, and enumerating superspecial curves. In particular, the second algorithm provides an efficient tool to judge the superspeciality of an algebraic curve.

<sup>1</sup>Kobe City College of Technology

## Facial Reduction for Exact Polynomial Sum of Squares Decompositions

Santiago Laplagne<sup>1</sup>

18J  
17:10  
B16

We develop new tools for decomposing a non-negative polynomial as an exact sum of squares (SOS) in the case where the associated semidefinite program is feasible but not strictly feasible (for example if the polynomial has real zeros). Computing symbolically

roots of the original polynomial and applying facial reduction techniques, we can solve the problem algebraically or restrict to a subspace where the problem becomes strictly feasible and a numerical approximation can be rounded to an exact solution. Our motivation for studying this problem is to determine when can a rational polynomial that is a sum of squares of polynomials with real coefficients be written as sum of squares of polynomials with rational coefficients. Applying our new strategy we can answer this question for some previously unknown cases. We first prove that if  $f$  is the sum of two squares with coefficients in an algebraic extension of  $\mathbb{Q}$  of odd degree, then it can always be decomposed as a rational SOS. For the case of more than two polynomials we provide an example of an irreducible polynomial that is the sum of three squares with coefficients in  $\mathbb{Q}(\sqrt[3]{2})$  that cannot be decomposed as a rational SOS.

<sup>1</sup>Universidad de Buenos Aires

## **Toric degeneration of Schubert varieties from matching fields**

**Oliver Clarke<sup>1</sup>, Fatemeh Mohammadi<sup>1</sup>**

Cancelled

We study toric degenerations of Schubert varieties inside Grassmannians  $\text{Gr}(k, n)$  and flag varieties  $\text{Fl}_n$  embedded in a product of Grassmannians. We provide a family of ideals whose combinatorics is governed by permutations in  $S_n$  and by matching fields in the sense of Sturmfels-Zelevinsky. We prove that these ideals are all quadratically generated and we study the toric ideals among them. We give a complete characterization of such toric ideals in terms of the combinatorics of matching fields and permutations.

<sup>1</sup>University of Bristol

## **Geometric envelopes using the Gröbner Cover**

**Antonio Montes<sup>1</sup>**

21J

12:20

S118

The Gröbner Cover is the method for the canonical discussion of parametric polynomial systems. It provides new resources for many applications. Particularly, the recently published book “The Gröbner Cover” develops three applications: Automatic deduction of geometric theorems, Geometric loci and Geometric envelopes.

The talk reviews the new ideas included in the book concerning envelopes, summarizing definitions and theorems, and develops an illustrative example that was left as an open problem in the book and for which we present new results.

<sup>1</sup>Universitat Politècnica de Catalunya, Spain

## **A new general formula to compute the Cauchy index with subresultants in an interval**

**Daniel Perrucci<sup>1</sup>, Marie-Françoise Roy<sup>2</sup>**

17J

16:10

B15

We present a new formula to compute the Cauchy index of a rational function in an interval using subresultant polynomials. There is no condition on the endpoints of the interval and the formula also involves in some cases less subresultant polynomials.

<sup>1</sup>Universidad de Buenos Aires, Argentina

<sup>2</sup>Université de Rennes 1, France

## On the rigidity of toric varieties associated to bipartite graphs

19J  
11:40  
B16

Irem Portakal<sup>1</sup>

One can associate to a bipartite graph a so-called edge ring and its spectrum is an affine normal toric variety. We first characterize the faces of the (edge) cone associated to this toric variety in terms of certain independent sets of the bipartite graph. We observe certain interesting classes of rigid toric varieties, (e.g. not isolated or not  $\mathbb{Q}$ -Gorenstein) and that their rigidity can be proven with our methods purely in terms of graphs. Moreover, we study torus actions on matrix Schubert varieties. In the toric case, it turns out that these varieties are a subclass of the toric varieties arising from bipartite graphs. Hence we present a classification for rigid toric matrix Schubert varieties.

<sup>1</sup>Otto-von-Guericke-Universität Magdeburg

## Transversal Intersection and Sum of Polynomial Ideals

21J  
11:40  
S118

Joydip Saha<sup>1</sup>, Indranath Sengupta<sup>1</sup>, Gaurab Tripathi<sup>2</sup>

In this talk we derive some conditions for transversal intersection of polynomial ideals. We exhibit some examples. Finally, as an application of the results proved, we compute the Betti numbers for ideals of the form  $I_1(XY) + J$ , where  $X$  and  $Y$  are matrices and  $J$  is the ideal generated by the  $2 \times 2$  minors of the matrix consisting of any two rows of  $X$ .

<sup>1</sup>IIT Gandhinagar

<sup>2</sup>Jadavpur University

## Counting realizations of Laman graphs on the sphere

17J  
12:10  
B16

Josef Schicho<sup>1</sup>, Matteo Gallet<sup>2</sup>, Georg Grasegger<sup>3</sup>

We present an algorithm that computes the number of realizations of a Laman graph on a sphere for a general choice of the angles between the vertices. The algorithm is based on the interpretation of such a realization as a point in the moduli space of stable curves of genus zero with marked points, and on the explicit description, due to Keel, of the Chow ring of this space.

<sup>1</sup>Johannes Kepler University, Linz, Austria

<sup>2</sup>International School for Advanced Studies, Trieste, Italy

<sup>3</sup>Research Institute for Computational and Applied Mathematics, Linz, Austria

## An Experimental Comparison of SONC and SOS Certificates for Unconstrained Optimization

Henning Seidler<sup>1</sup>, Timo de Wolff<sup>1</sup>

Finding the minimum of a multivariate real polynomial is a well-known hard problem with various applications. We present a polynomial time algorithm to approximate such lower bounds via sums of nonnegative circuit polynomials (SONC). As a main result, we carry out the first large-scale comparison of SONC, using this algorithm and different geometric programming (GP) solvers, with the classical sums of squares (SOS) approach, using several of the most common semidefinite programming (SDP) solvers. SONC yields bounds competitive to SOS in several cases, but using significantly less time and memory. In particular, SONC/GP can handle much larger problem instances than SOS/SDP.

<sup>1</sup>Technische Universität Berlin

## A tensor version of the quantum Wielandt inequality

Tim Seynnaeve<sup>1</sup>, Mateusz Michałek<sup>2</sup>, Frank Verstraete<sup>3</sup>

The quantum Wielandt inequality, first proven by Sanz, Perez-Garcia, Wolf and Cirac, can be stated as follows:

Let  $L$  be a linear space of  $D \times D$ -matrices, and let  $L^k$  be the linear space spanned by all products of exactly  $k$  matrices in  $L$ . There is a constant  $C(D)$ , only depending on  $D$ , such that if  $\dim(L^k) = D^2$  for some  $k$ , then it also holds for all  $k \leq C(D)$ . Moreover,  $C(D) = \mathcal{O}(D^4)$ .

This theorem was motivated by the study of Matrix Product States in quantum information theory. The bound on  $C(D)$  was recently improved to  $\mathcal{O}(D^2 \log D)$ . Both bounds were obtained using explicit methods from linear algebra. In this talk we discuss a generalization of the quantum Wielandt theorem, where the matrices are replaced by tensors on an  $n$ -dimensional grid. Our main new insight is the application of nonconstructive Noetherian arguments from non-linear algebra. Our main motivations are applications to Projected Entangled Pair States (PEPS). These are higher dimensional versions of Matrix Product States, for which none of the above bounds were known.

<sup>1</sup>Max Planck Institute for Mathematics in the Sciences, Leipzig

<sup>2</sup>Max Planck Institute for Mathematics in the Sciences, Leipzig, Polish Academy of Sciences, and Aalto University

<sup>3</sup>Ghent University

## Unexpected hypersurfaces with multiple fat points

Justyna Szpond<sup>1</sup>

18J  
17:50  
B16

20J  
12:20  
B15

18J  
16:10  
B15

Starting with the ground-breaking work of Cook II, Harbourne, Migliore and Nagel, there has been a lot of interest in unexpected hypersurfaces. In the last couple of months a considerable number of new examples and new phenomena has been observed and reported on. All examples studied so far had just one fat point. In this note we introduce a new series of examples, which establishes for the first time the existence of unexpected hypersurfaces with multiple fat points. The key underlying idea is to study Fermat-type configurations of points in projective spaces.

<sup>1</sup>University of Cracow

## Voronoi Cells of Varieties

21J

11:40

B16

**Madeleine Weinstein<sup>1</sup>, Bernd Sturmfels<sup>2</sup>, Diego Cifuentes<sup>3</sup>, Kristian Ranestad<sup>4</sup>**

Every real algebraic variety determines a Voronoi decomposition of its ambient Euclidean space. Each Voronoi cell is a convex semialgebraic set in the normal space of the variety at a point. We compute the algebraic boundaries of these Voronoi cells. Using intersection theory, we give a formula for the degrees of the algebraic boundaries of Voronoi cells of curves and surfaces. We discuss an application to low-rank matrix approximation.

<sup>1</sup>UC Berkeley

<sup>2</sup>MPI-MiS Leipzig and UC Berkeley

<sup>3</sup>Massachusetts Institute of Technology

<sup>4</sup>University of Oslo



SOFTWARE PRESENTATIONS

## The 144 symmetries of the Littlewood-Richardson coefficients of $SL_3$

Emmanuel Briand<sup>1</sup>, Mercedes Rosas<sup>1</sup>,

We compute with SageMath the group of all linear symmetries for the Littlewood-Richardson associated to the representations of  $SL_3$ . We find that there are 144 symmetries, more than the 12 symmetries known for the Littlewood-Richardson coefficients in general.

<sup>1</sup>Universidad de Sevilla

20J  
16:20  
B16

## Computing the distance to the positive part of phylogenetic varieties

Marta Casanellas<sup>1</sup>, Jesús Fernández Sánchez<sup>1</sup>, Marina Garrote López<sup>2</sup>,

Evolution can be modelled adopting parametric statistical models which allow to define a joint probability distribution at the leaves of phylogenetic trees. When these models are algebraic, one is able to deduce polynomial relationships between these probabilities, and the study of these polynomials and the geometry of the algebraic varieties that arise from them can be used to reconstruct phylogenetic trees. However, not all points in these algebraic varieties have biological sense. In this talk, we would like to explore the extent to which restricting to the subset of these varieties with biological sense subsets can provide insight into existent methods of phylogenetic reconstruction. The projection into these subsets can be seen as an optimization problem and can be solved using nonlinear programming algorithms. As these algorithms do not guarantee a global solution, we use a different approach that allows us to find a global optimum. Numerical algebraic geometry and computational algebra play a fundamental role here. We work with trees evolving under groups-based models and, in particular, we explore the long branch attraction phenomena.

<sup>1</sup>Universitat Politècnica de Catalunya

<sup>2</sup>Universitat Politècnica de Catalunya–BGSM

20J  
17:30  
B16

## Computing with Kenzo from Sage

J. Cuevas-Rozo<sup>1</sup>, M. A. Marco Buzunáriz<sup>1</sup>, A. Romero<sup>1</sup>,

In this paper we present an interface between SageMath and Kenzo, together with an optional Kenzo package. Our work makes it possible to communicate both computer algebra programs and enhances the Sage- Math system with new capabilities in algebraic topology, dealing in particular with simplicial objects of infinite nature.

20J  
17:30  
B15

<sup>1</sup>Universidad de Zaragoza

## What's new on Maple?

20J  
15:00

**John May**<sup>1</sup>

S118 This is a special talk dedicated to illustrate the new features of Maple.

<sup>1</sup>Maplesoft

## Effective computation of degree bounded Minimal models of GCDA's

20J  
18:10

**Victor Manero**<sup>1</sup>, **Miguel Ángel Marco Buzunáriz**<sup>1</sup>,

B15 Given a finitely presented Graded Commutative Differential Algebra (GCDA), we present a method to compute its minimal model, together with a map that is a quasi-isomorphism up to a given degree. The method works by adding generators one by one. We also provide a specific implementation of the method.

<sup>1</sup>Universidad de Zaragoza

## Current status of GeoGebra Automated Reasoning Tools

20J  
16:20

**M. Pilar Vélez**<sup>1</sup>, **Tomás Recio**<sup>2</sup>,

B15 The presentation attempts to introduce, describe and exemplify the technical features of some recently implemented automated reasoning tools (for mechanically finding relations among geometric elements, for testing the truth or falsity of some statement, for finding additional hypotheses for a given statement to hold) in the dynamic mathematics software GeoGebra and that have been awarded the Software Demo Award at ISSAC 2016. These tools are based on symbolic computation algorithms, allowing the automatic and rigorous proving and discovery of theorems on constructed geometric figures. It would also present a description of the current state of development, including the "truth on parts" test, already implemented in GeoGebra; and the "automated geometer" web prototype.

<sup>1</sup>Universidad de Cantabria

<sup>2</sup>Universidad de Nebrija



LIST OF CONTRIBUTED POSTERS

Michela Ceria	<i>Combinatorial decompositions for monomial ideals</i>
Momonari Kudo, Shushi Harashita	<i>Superspecial trigonal curves of genus 5</i>
Andre Galligo, Zbigniew Jelonek	<i>Elimination ideals and Bezout relations</i>
Patricio Almirón and Guillem Blanco	<i>On the Tjurina number of plane curve singularities</i>
Martin Vodicka	<i>Normality of the Kimura 3-parameter model</i>
Alexandru Iosif and Carsten Conradi and Thomas Kahle	<i>Multistationarity in the space of total concentrations for systems that admit a monomial parametrization</i>
Josep Alvarez Montaner, Alberto Fernández Boix and Santiago Zarzuela	<i>A Koszul complex over skew polynomial rings</i>
Carlos R. Guzmán and Miguel Robredo	<i>On multiplier ideals of plane curve singularities</i>
Miruna-Stefana Sorea	<i>Construction of separable Poincare-Reeb trees</i>
Lamprini Ananiadi and Eliana Duarte	<i>Gröbner Bases for Toric Staged Trees</i>
Alberto Llorente	<i>Improvements in the complexity of the algorithms for computing the Liouvillian solutions of differential equations</i>
Simon Telen, Marc Van Barel and Jan Verschelde	<i>Robust Numerical Path Tracking for Polynomial Homotopies</i>
Francisco Criado, Michael Joswig and Francisco Santos	<i>Tropical Voronoi Diagrams</i>
Cristina Bertone	<i>Equations, Singularities, Irreducible components of Hilbert and Quot schemes</i>
Tristram Bogart and Juan C. Torres	<i>Projectively Unique Order Polytopes</i>
Luca Sodomaco	<i>The Distance Function from the Variety of rank-one partially symmetric Tensors</i>
Francisco-Jesús Castro-Jimenez, María-Emilia Alonso and Herwig Hauser	<i>Encoding Algebraic Power Series</i>
Peter Bürgisser, Felipe Cucker and Josue Tonelli Cueto	<i>Computing the Homology Groups of Closed Semialgebraic Sets in Weak Exponential Time</i>
Carla Mascia, Giancarlo Rinaldo and Francesco Romeo	<i>On the primality of one-hole polyomino ideals</i>
Oguzhan Yürük, Timo de Wolff and Olivia Röhrig	<i>Initial Steps in the Classification of Maximal Mediated Sets</i>
Matthew England and Dorian Florescu	<i>Comparing machine learning models to choose the variable ordering for cylindrical algebraic decomposition</i>

Henryk Nagel and Yue Ren	<i>Hands-on tropical geometry</i>
Beatriz Pascual-Escudero	<i>Singularities in the visual servoing of four image points: a study based on the hidden robot model</i>
Roberto Alvarenga	<i>On the Katz's bound and Artin-Schreier curves</i>
Grigoriy Blekherman and Jaewoo Jung	<i>Bounds of the regularity of quadratic monomial ideals</i>
Margaret Regan	<i>Real Monodromy Action</i>
Daniel Irving Bernstein, Grigoriy Blekherman and Kisun Lee	<i>Typical ranks of semisimple graphs</i>

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